Numerical Problems: B-Splines, Rational Curves & Surfaces

B.Tech Engineering Coursework - Step-by-Step Solutions

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Problem 1: Basis Function Calculation

Problem Statement: Given a knot vector $U = \{0, 0, 1, 2, 2\}$ and degree $p = 1$. Calculate the values of all non-zero basis functions at the parameter value $u = 0.5$.

1. Identify the Interval

The parameter $u = 0.5$ lies in the knot span $[u_1, u_2)$, which is $[0, 1)$.

2. Zeroth Degree Basis ($p = 0$)

By definition, $N_{i,0}(u) = 1$ if $u_i \leq u < u_{i+1}$.
Since $0.5 \in [u_1, u_2)$, then $N_{1,0}(0.5) = 1$. All other $N_{i,0} = 0$.

Problem 1: Continued (Recursion)

Using the Cox-de Boor formula for $p = 1$:

$$N_{i,1}(u) = \frac{u - u_i}{u_{i+1} - u_i} N_{i,0}(u) + \frac{u_{i+2} - u}{u_{i+2} - u_{i+1}} N_{i+1,0}(u)$$

For $i = 0$**:**

$$N_{0,1}(0.5) = \frac{0.5 - 0}{0 - 0}(0) + \frac{1 - 0.5}{1 - 0}(1) = 0 + 0.5 = 0.5$$

For $i = 1$**:**

$$N_{1,1}(0.5) = \frac{0.5 - 0}{1 - 0}(1) + \frac{2 - 0.5}{2 - 1}(0) = 0.5 + 0 = 0.5$$

Check: $N_{0,1} + N_{1,1} = 1.0$ *(Correct)*

Problem 2: NURBS Point Evaluation

Problem Statement: A quadratic Rational Bézier curve has:

- Control Points: $\mathrm{P}_0(0,0), \mathrm{P}_1(2,4), \mathrm{P}_2(4,0)$
- Weights: $w_0 = 1, w_1 = 2, w_2 = 1$

Find the coordinates of the curve at $u = 0.5$.

Step 1: Calculate Bernstein Polynomials (Basis)

At $u = 0.5$ for degree $p = 2$:

- $B_{0,2} = (1 - 0.5)^2 = 0.25$
- $B_{1,2} = 2(0.5)(1 - 0.5) = 0.50$
- $B_{2,2} = (0.5)^2 = 0.25$

Problem 2: Continued (Weighting)

Step 2: Calculate Denominator (Total Weight)

$$D = \sum w_i B_i = (1 \times 0.25) + (2 \times 0.50) + (1 \times 0.25) = 0.25 + 1.0 + 0.25 = 1.5$$

Step 3: Calculate Numerator (Weighted Points)

$$X_{num} = (1 \cdot 0.25 \cdot 0) + (2 \cdot 0.50 \cdot 2) + (1 \cdot 0.25 \cdot 4) = 0 + 2 + 1 = 3$$

$$Y_{num} = (1 \cdot 0.25 \cdot 0) + (2 \cdot 0.50 \cdot 4) + (1 \cdot 0.25 \cdot 0) = 0 + 4 + 0 = 4$$

Step 4: Final Division

$$\mathrm{C}(0.5) = \left(\frac{3}{1.5}, \frac{4}{1.5}\right) = (2, 2.67)$$

Problem 3: Bilinear Surface Patch

Problem Statement: Find the 3D point at $u = 0.5, v = 0.5$ for a surface patch with: $\mathrm{P}_{0,0}(0,0,0), \mathrm{P}_{1,0}(10,0,0), \mathrm{P}_{0,1}(0,10,5), \mathrm{P}_{1,1}(10,10,0)$.

The Bilinear Equation

$$\mathrm{S}(u,v) = (1-u)(1-v)\mathrm{P}_{0,0} + u(1-v)\mathrm{P}_{1,0} + (1-u)v\mathrm{P}_{0,1} + uv\mathrm{P}_{1,1}$$

Substitution for $u = 0.5, v = 0.5$: All coefficients $(1-0.5)(1-0.5)$ etc. $\mathbf{= 0.25}$.

Problem 3: Final Calculation

Summing the components:

- $X = 0.25(0 + 10 + 0 + 10) = 0.25(20) = 5$
- $Y = 0.25(0 + 0 + 10 + 10) = 0.25(20) = 5$
- $Z = 0.25(0 + 0 + 5 + 0) = 0.25(5) = 1.25$

Final Coordinate

The point on the surface is $(5, 5, 1.25)$.

Observation: Note how the Z-value is pulled toward the only non-zero height point $\mathrm{P}_{0,1}$.

Summary of Formulas for Exam

- **B-Spline:** $\mathrm{C}(u) = \sum N_{i,p}\mathrm{P}_i$
- **Knot Relation:** $m = n + p + 1$
- **NURBS:** $\mathrm{C}(u) = \frac{\sum w_i N_i \mathrm{P}_i}{\sum w_i N_i}$
- **Tensor Surface:** $\mathrm{S}(u, v) = \sum \sum N_i M_j \mathrm{P}_{i,j}$