Principles of Elasticity and Introduction to Numerical Methods

A Comprehensive Introduction for Beginners

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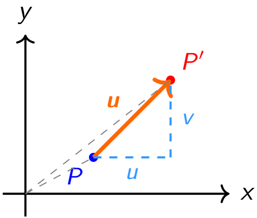
Course Overview

1. Strain-Displacement Relations
2. Stress-Strain Relations
3. Plane Stress and Plane Strain
4. Introduction to Numerical Methods
5. Potential Energy Method
6. Rayleigh-Ritz Method
7. Galerkin Method
8. Summary and Practice

What is Displacement?

Definition:

- Change in position of a point in a body
- Vector quantity with magnitude and direction
- Denoted by u, v, w in x, y, z directions



Displacement Vector:

$$\boldsymbol{u} = u\,\boldsymbol{i} + v\,\boldsymbol{j} + w\,\boldsymbol{k}$$

where:

- u = displacement in x-direction
- v = displacement in y-direction
- w = displacement in z-direction

What is Strain?

Definition of Strain

Strain is the **relative displacement** or **deformation per unit length** in a body due to applied forces.

Types of Strain:

1. **Normal Strain** (ε)
   - Change in length per unit length
   - $\varepsilon = \frac{\Delta L}{L}$
2. **Shear Strain** (γ)
   - Change in angle (distortion)
   - $\gamma \approx \theta$ (small angles)

Characteristics:

- Normal strain: elongation or compression
- Shear strain: angular distortion
- Both are dimensionless
- Measured as ratio or percentage

Strain-Displacement Relations in 1D

For a one-dimensional bar:

Consider an infinitesimal element of length dx at position x.

Normal Strain:

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

where:

- ε_x = normal strain in x-direction
- u = displacement in x-direction
- $\frac{\partial u}{\partial x}$ = rate of change of displacement

Physical Meaning:

- If $\varepsilon_x > 0 \rightarrow$ Elongation (tension)
- If $\varepsilon_x < 0 \rightarrow$ Contraction (compression)

Example:

Original length: dx
After deformation:
$dx + \frac{\partial u}{\partial x} dx$

Change in length:
$\frac{\partial u}{\partial x} dx$

Strain: $\varepsilon_x = \frac{\partial u}{\partial x}$

Strain-Displacement Relations in 2D

For a two-dimensional element:

Normal Strains:

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y}$$

Shear Strain:

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

Tensor Shear Strain:

$$\varepsilon_{xy} = \frac{1}{2}\gamma_{xy}$$

Total of 3 strain components:

- ε_x - normal strain in x
- ε_y - normal strain in y
- γ_{xy} - shear strain in xy-plane

Note:

- All strains are dimensionless
- Functions of displacement gradients
- Form a symmetric tensor

Strain-Displacement Relations in 3D

For a three-dimensional element:

Complete 3D Strain-Displacement Relations

Normal Strains:

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}$$

Shear Strains:

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

Strain Tensor (Matrix Form):

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_z \end{bmatrix}$$

Note: Symmetric tensor ($\varepsilon_{ij} = \varepsilon_{ji}$) with **6 independent components** in 3D

Summary: Strain-Displacement Relations

Dimension	**Normal Strains**	**Shear Strains**
1D	$\varepsilon_x = \frac{\partial u}{\partial x}$	None
2D	$\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}$	$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$
3D	$\varepsilon_x, \varepsilon_y, \varepsilon_z$	$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$

Key Points

- Strain relates deformation to displacement gradients
- Normal strains: elongation/contraction along axes
- Shear strains: distortion (change in angles)
- Total of **6 independent strain components** in 3D

What is Stress?

Definition:

- Internal force per unit area
- Measured in Pa (Pascal) or MPa
- Arises due to external loads

Types:

1. **Normal Stress** (σ)
   - Perpendicular to surface
   - Tension (+) or Compression (-)
   - $\sigma = F/A$
2. **Shear Stress** (τ)
   - Parallel to surface
   - Causes sliding/distortion
   - $\tau = F/A$

Stress Tensor in 3D:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Total of **6 independent stress components** (symmetric tensor)

Hooke's Law: Foundation

Hooke's Law

For **linear elastic materials**, stress is **directly proportional** to strain within elastic limit.

1D Hooke's Law:

$$\sigma = E\varepsilon$$

where: σ = normal stress (Pa), E = Young's modulus (Pa), ε = normal strain

For Shear:

$$\tau = G\gamma$$

where: $G = \frac{E}{2(1+\nu)}$ is the shear modulus

Typical Values of E:

Material	E (GPa)
Steel	200
Aluminum	70
Copper	120
Concrete	30
Wood	10

Material Properties: Poisson's Ratio

Definition:
When a material is stretched in one direction, it contracts in perpendicular directions.

Poisson's Ratio:

$$\nu = -\frac{\varepsilon_{\text{lateral}}}{\varepsilon_{\text{axial}}}$$

Example: Rod stretched along x-axis:

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\varepsilon_y = \varepsilon_z = -\nu \frac{\sigma_x}{E}$$

Typical Values:

- Steel: $\nu \approx 0.3$
- Aluminum: $\nu \approx 0.33$

Physical Meaning:

- Negative sign: lateral strain opposite to axial
- Range: $0 \leq \nu \leq 0.5$
- Most materials: $0.25 - 0.35$
- $\nu = 0.5$: incompressible

Formula:

$$\nu = -\frac{\Delta D / D}{\Delta L / L}$$

where $\Delta L =$ change in length, $\Delta D =$ change in diameter

Stress-Strain Relations for 1D Case

For uniaxial loading (e.g., a bar under tension):

Given: Stress σ_x in x-direction

Strains:

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E}$$

Inverse:

$$\sigma_x = E\varepsilon_x$$

For Shear:

$$\tau_{xy} = G\gamma_{xy}$$

Example:

Steel bar: $E = 200$ GPa, $\nu = 0.3$, $L = 2$ m, $A = 100$ mm^2, $P = 10$ kN

Solution:

$$\sigma_x = \frac{P}{A} = \frac{10 \times 10^3}{100 \times 10^{-6}}$$
$$= 100 \text{ MPa}$$

$$\varepsilon_x = \frac{100}{200 \times 10^3} = 5 \times 10^{-4}$$

$$\Delta L = 5 \times 10^{-4} \times 2 = 1 \text{ mm}$$

Stress-Strain Relations for 2D Case

For biaxial stress state (σ_x, σ_y, τ_{xy}):

Generalized Hooke's Law in 2D

Strain in terms of Stress:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y), \quad \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x), \quad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

Stress in terms of Strain:

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y), \quad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x), \quad \tau_{xy} = G\gamma_{xy}$$

Stress-Strain Relations for 3D Case

For general 3D stress state:

Generalized Hooke's Law in 3D

Strain in terms of Stress:

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)], \quad \varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$
$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)], \quad \gamma_{ij} = \frac{\tau_{ij}}{G}$$

Compact Matrix Form:

$$\{\varepsilon\} = [C]\{\sigma\}$$

where $[C]$ is the compliance matrix (6×6) depending on E and ν.

Material Constants Relationship

Elastic constants are interrelated:

Key Relationships

1. **Shear Modulus:** $G = \frac{E}{2(1+\nu)}$
2. **Bulk Modulus:** $K = \frac{E}{3(1-2\nu)}$
3. **Lamé's Constants:** $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$, $\mu = G$

Independent Constants:

For **isotropic** materials, only **2 independent constants** needed: commonly E and ν

Introduction to 2D Simplifications

Why 2D Analysis?

- Reduces computational effort
- Easier to visualize and solve
- Accurate for specific geometries

Two Main Cases:

1. **Plane Stress –** Thin structures (plates, shells)
2. **Plane Strain –** Long structures (dams, tunnels, pipes)

Key Difference

- **Plane Stress:** Out-of-plane **stresses** are zero
- **Plane Strain:** Out-of-plane **strains** are zero

Plane Stress Conditions

Definition: Stress components perpendicular to plane are zero.

Assumptions:

- Thin plate: $t \ll L, W$
- Loading in xy-plane only
- No loads on top/bottom surfaces

Stress State:

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$

Non-zero: $\sigma_x, \sigma_y, \tau_{xy}$

Examples:

- Thin pressure vessels
- Sheet metal forming
- Thin-walled tubes

Applications:

- Aerospace structures (thin skins)
- Automotive body panels
- Storage tanks
- Thin plates with in-plane loading

Criterion:

$$t < \frac{L}{10} \text{ or } \frac{W}{10}$$

Plane Stress: Stress-Strain Relations

For Plane Stress conditions:

Constitutive Relations

Strain in terms of Stress:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y), \quad \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$
$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) \neq 0, \quad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

Stress in terms of Strain:

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y), \quad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

Note: Even though $\sigma_z = 0$, strain $\varepsilon_z \neq 0$ (plate gets thinner/thicker)

Plane Strain Conditions

Definition: Strain components perpendicular to plane are zero.

Assumptions:

- Long structure: $L \gg W, t$
- Deformation constrained in z
- Uniform cross-section

Strain State:

$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$$

Non-zero: $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

Examples:

- Long retaining walls
- Tunnels and pipelines
- Dams, embankments

Applications:

- Underground structures
- Long prismatic sections
- Railway tunnels
- Long pipelines under pressure

Criterion:

$$L > 10 \times W \text{ or } 10 \times t$$

Plane Strain: Stress-Strain Relations

For Plane Strain conditions:

Constitutive Relations

Strain in terms of Stress:

$$\varepsilon_x = \frac{1+\nu}{E}[(1-\nu)\sigma_x - \nu\sigma_y], \quad \varepsilon_y = \frac{1+\nu}{E}[(1-\nu)\sigma_y - \nu\sigma_x]$$
$$\sigma_z = \nu(\sigma_x + \sigma_y) \neq 0, \quad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

Stress in terms of Strain:

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_x + \nu\varepsilon_y]$$
$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_y + \nu\varepsilon_x]$$

Note: Even though $\varepsilon_z = 0$, stress $\sigma_z \neq 0$ (constraint generates stress)

Comparison: Plane Stress vs Plane Strain

Aspect	**Plane Stress**	**Plane Strain**
Geometry	Thin: $t \ll L, W$	Long: $L \gg W, t$
Zero	$\sigma_z = \tau_{xz} = \tau_{yz} = 0$	$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$
Out-of-plane	$\varepsilon_z \neq 0$	$\sigma_z \neq 0$
Examples	Pressure vessels, sheet metal	Tunnels, dams, long pipes

Key Insight

- **Plane Stress:** Out-of-plane direction is *free* (can deform)
- **Plane Strain:** Out-of-plane direction is *constrained* (cannot deform)

Why Numerical Methods?

Limitations of Analytical Solutions:

- Complex geometries (irregular shapes)
- Non-uniform material properties
- Complicated boundary conditions
- Variable loading patterns

Numerical Methods Provide:

1. Approximate solutions to complex problems
2. Solutions where exact analytical methods fail
3. Ability to handle real-world engineering problems
4. Foundation for Finite Element Analysis (FEA)

Common Numerical Methods

- Potential Energy Method

Classification

1. **Variational Methods (Energy-Based):**
    - Potential Energy Method
    - Rayleigh-Ritz Method
    - Based on minimizing total energy
2. **Weighted Residual Methods (Equation-Based):**
    - Galerkin Method
    - Collocation Method
    - Based on minimizing error in governing equation

Fundamental Concepts

Key concepts for numerical methods:

1. **Trial Function:**
    - Assumed displacement field with unknown parameters
    - Must satisfy boundary conditions
2. **Energy Functionals:**
    - Mathematical expressions for total energy
    - Must be minimized for equilibrium
3. **Residual:**
    - Error when approximate solution substituted in governing equation
    - $R = \mathcal{L}(\tilde{u}) - f$
4. **Degrees of Freedom (DOF):**
    - Unknown coefficients in trial function
    - More DOF → Better approximation

Principle of Minimum Potential Energy

Principle Statement

Of all displacement fields satisfying boundary conditions, the **actual displacement field** makes the **total potential energy minimum**.

Total Potential Energy:

$$\Pi = U - W$$

where: Π = Total potential energy, U = Strain energy, W = Work done

Equilibrium Condition:

$$\frac{\partial \Pi}{\partial q_i} = 0 \quad \text{for all DOF } q_i$$

This gives a system of equations to solve for unknown coefficients.

Strain Energy for Different Cases

1. **Axially Loaded Bar:**

$$U = \int_0^L \frac{1}{2} EA \left(\frac{du}{dx} \right)^2 dx$$

2. **Beam in Bending:**

$$U = \int_0^L \frac{1}{2} EI \left(\frac{d^2 w}{dx^2} \right)^2 dx$$

3. **General 3D Body:**

$$U = \frac{1}{2} \int_V \boldsymbol{\sigma}^T \boldsymbol{\varepsilon} \, dV$$

where: A = area, E = Young's modulus, I = moment of inertia, w = deflection

Work Done by External Forces

For different loading types:

1. **Concentrated Force P at $x = a$:**

$$W = P \cdot u(a)$$

2. **Distributed Load $q(x)$ along bar:**

$$W = \int_0^L q(x) \cdot u(x)\, dx$$

3. **For Beam with transverse load $p(x)$:**

$$W = \int_0^L p(x) \cdot w(x)\, dx + M \cdot \theta$$

where: $u(x)$ = axial displacement, $w(x)$ = transverse deflection, M = moment, θ = rotation

Example 1: Axially Loaded Bar

Problem: Bar: length L, area A, modulus E, fixed at $x = 0$, force P at $x = L$

Step 1: Assume $u(x) = a_1 x$

Step 2: Strain Energy

$$\begin{aligned} U &= \int_0^L \frac{1}{2} EA a_1^2 dx \\ &= \frac{1}{2} EA a_1^2 L \end{aligned}$$

Step 3: Work Done

$$W = P \cdot a_1 L$$

Step 4: Total Potential Energy

$$\Pi = \frac{1}{2} EA a_1^2 L - P a_1 L$$

Step 5: Minimize

$$\begin{aligned} \frac{\partial \Pi}{\partial a_1} &= EA a_1 L - PL = 0 \\ a_1 &= \frac{P}{EA} \end{aligned}$$

Solution:

$$\boxed{u(x) = \frac{P}{EA} x}$$

Exact match with analytical solution!

Rayleigh-Ritz Method: Overview

Method Description

Extension of potential energy method using **series of trial functions** with multiple unknown coefficients.

Key Features:

- Uses linear combination of basis functions
- More flexible than single-term approximation
- Accuracy improves with more terms
- Foundation for Finite Element Method

General Form:

$$u(x) = \sum_{i=1}^{n} a_i \phi_i(x)$$

where: $\phi_i(x)$ = basis functions (known), a_i = unknown coefficients, n = number of terms

Selection of Basis Functions

Requirements for $\phi_i(x)$:

1. **Satisfy essential boundary conditions**
2. **Linearly independent**
3. **Complete set**

Common Choices:

Type	**Example**	**Application**
Polynomial	$\phi_i(x) = x^i$	Simple problems
Trigonometric	$\phi_i(x) = \sin(i\pi x/L)$	Periodic problems
Combined	$\phi_i(x) = x^i(L - x)$	Satisfying BCs

Rayleigh-Ritz Procedure

Step-by-step procedure:

1. Assume trial function: $u(x) = \sum_{i=1}^{n} a_i \phi_i(x)$
2. Calculate strain energy $U = U(a_1, a_2, \dots, a_n)$
3. Calculate work done $W = W(a_1, a_2, \dots, a_n)$
4. Form total potential energy: $\Pi = U - W$
5. Minimize: $\frac{\partial \Pi}{\partial a_i} = 0,\ i = 1, 2, \dots, n$
6. Solve system of n equations for $a_1, a_2, \dots, a_n$

Example 2: Cantilever Beam

Problem: Cantilever, length L, EI constant, point load P at free end
BCs: At $x = 0$: $w = 0$, $\frac{dw}{dx} = 0$
Trial: $w(x) = a_1 x^2 + a_2 x^3$

Strain Energy:

$$U = \frac{EI}{2}\left[4a_1^2 L + 12a_1 a_2 L^2 + 12a_2^2 L^3\right]$$

Work: $W = P(a_1 L^2 + a_2 L^3)$
Minimize:

$$\frac{\partial \Pi}{\partial a_1} = 0, \quad \frac{\partial \Pi}{\partial a_2} = 0$$

Solution: $a_1 = \frac{P}{2EI}$, $a_2 = -\frac{P}{6EIL}$

$$\boxed{w(x) = \frac{Px^2}{6EIL}(3L - x)}$$

Exact match!

Example 3: Simply Supported Beam

Problem: Simply supported, length L, EI constant, uniform load q
BCs: $w(0) = 0$, $w(L) = 0$
Trial: $w(x) = a_1 x(L - x)$

Strain Energy:

$$U = 2EIa_1^2 L$$

Work:

$$W = qa_1 \frac{L^3}{6}$$

Minimize:

$$\frac{\partial \Pi}{\partial a_1} = 0$$

$$a_1 = \frac{qL^2}{24EI}$$

Solution:

$$w(x) = \frac{qL^2}{24EI} x(L - x)$$

Max deflection: $w_{\text{max}} = \frac{qL^4}{96EI}$
Error 25% (1-term approx.)

Galerkin Method: Introduction

Method Description

A **weighted residual method** that minimizes error (residual) in governing differential equation using weight functions.

Key Difference:

- Rayleigh-Ritz: Energy-based (requires energy functional)
- Galerkin: Equation-based (works directly with differential equation)

Advantages:

- More general applicability
- Works with any differential equation
- Foundation for Finite Element Method

Concept: For equation $\mathcal{L}(u) = f$, approximate solution $\tilde{u}$ gives residual $R = \mathcal{L}(\tilde{u}) - f$. Force weighted average of residual to zero.

Galerkin Method: Formulation

Governing Equation:

$$\mathcal{L}(u) = f(x) \quad \text{in domain } \Omega$$

Trial Solution:

$$\tilde{u}(x) = \sum_{i=1}^{n} a_i \phi_i(x)$$

Residual:

$$R(x) = \mathcal{L}(\tilde{u}) - f(x) \neq 0$$

Galerkin Condition:

$$\int_{\Omega} \phi_j(x) \cdot R(x)\, dx = 0, \quad j = 1, 2, \ldots, n$$

This gives n equations for n unknowns. Weight functions = basis functions (ϕ_j)

Galerkin Procedure

Step-by-step:

1. Write governing equation: $\mathcal{L}(u) = f(x)$
2. Assume trial solution: $\tilde{u}(x) = \sum a_i \phi_i(x)$
3. Get residual: $R(x) = \mathcal{L}(\tilde{u}) - f(x)$
4. Apply Galerkin: $\int_\Omega \phi_j \cdot R \, dx = 0$, $j = 1, \ldots, n$
5. Expand and simplify to get system of equations
6. Solve for coefficients $a_1, a_2, \ldots, a_n$

Example 4: Bar using Galerkin

Problem: Bar fixed at $x = 0$, force P at $x = L$, properties EA
Governing: $EA\frac{d^2u}{dx^2} = 0$
BCs: $u(0) = 0$, $EA\frac{du}{dx}\Big|_{x=L} = P$
Trial: $\tilde{u}(x) = a_1 x$

Residual: $R = EA \cdot 0 = 0$ everywhere
Using BC at $x = L$:

$$EA \cdot a_1 = P \quad \Rightarrow \quad a_1 = \frac{P}{EA}$$

Solution:

$$\boxed{u(x) = \frac{P}{EA}x}$$

Example 5: Beam using Galerkin

Problem: Simply supported beam, uniform load q
Governing: $EI\frac{d^4w}{dx^4} = q$
Trial: $w(x) = a_1 \sin\left(\frac{\pi x}{L}\right)$

Residual: $R = EI \cdot a_1 \frac{\pi^4}{L^4} \sin\left(\frac{\pi x}{L}\right) - q$
Galerkin: $\int_0^L \sin\left(\frac{\pi x}{L}\right) \cdot R\, dx = 0$
Using integrals: $\int_0^L \sin^2 = \frac{L}{2}$, $\int_0^L \sin = \frac{2L}{\pi}$

$$a_1 = \frac{4qL^4}{\pi^5 EI}$$

Solution:

$$\boxed{w(x) = \frac{4qL^4}{\pi^5 EI} \sin\left(\frac{\pi x}{L}\right)}$$

Comparison: Rayleigh-Ritz vs Galerkin

Aspect	**Rayleigh-Ritz**	**Galerkin**
Basis	Energy principles	Differential equation
Requires	Energy functional	Governing equation
Approach	Minimize energy	Minimize residual
Applicability	Energy formulation	Any equation

Key Insight

- For many structural problems, both methods give identical results
- Both are foundations for Finite Element Method

Summary: All Topics

1. **Strain-Displacement:** 6 independent components in 3D
2. **Stress-Strain:** Hooke's Law, 2 independent constants (E, ν)
3. **Plane Stress/Strain:** 2D simplifications for different geometries
4. **Numerical Methods:**

Method	**Key Feature**
Potential Energy	Minimize $\Pi = U - W$
Rayleigh-Ritz	$u = \sum a_i \phi_i(x)$
Galerkin	$\int \phi_j R \, dx = 0$

Practice Problem 1

Problem:

Cantilever: $L = 3$ m, $EI = 10^4$ N·m^2, uniform load $q = 1000$ N/m
Using Rayleigh-Ritz with $w(x) = a_1 x^2$, find:

1. Coefficient a_1
2. Maximum deflection
3. Compare with exact

Hints:

- $U = \frac{1}{2}\int_0^L EI\left(\frac{d^2w}{dx^2}\right)^2 dx$
- $W = \int_0^L q \cdot w(x) dx$
- Minimize: $\frac{\partial \Pi}{\partial a_1} = 0$

Practice Problem 2

Problem:
Bar: $L = 2$ m, $A = 500$ mm^2, $E = 200$ GPa
Distributed load: $p(x) = p_0 x/L$ (linearly varying)
Using Galerkin with $u(x) = a_1 x$, find:

1. Governing equation
2. Coefficient a_1 in terms of p_0
3. Displacement at free end

Hints:

- Governing: $EA\frac{d^2u}{dx^2} + p(x) = 0$
- Residual: $R = EA\frac{d^2u}{dx^2} + p(x)$
- Galerkin: $\int_0^L \phi_1 \cdot R\, dx = 0$

Key Takeaways

1. Strain-displacement relations connect kinematics to deformation
2. Stress-strain relations (Hooke's Law) connect forces to deformation
3. Plane stress/strain simplify 3D problems based on geometry
4. Numerical methods provide approximate solutions
5. Energy methods and weighted residual methods often equivalent
6. These form the foundation of Finite Element Analysis
7. Practice is essential to master concepts

Thank You!

Questions?

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All the best for your studies!