Principle of Transmissibility, Isolation Design, and Vibration Measurement

Advanced Mechanical Vibrations

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Outline of the Lecture

Course Outcomes

Course Outcomes (Based on Bloom's Taxonomy):

- **CO1 (Understand):** Explain the physics of free, forced, and damped vibrations across various mechanical systems.
- **CO2 (Apply):** Apply mathematical techniques to model and solve continuous vibration problems.
- **CO3 (Analyze):** Analyze the frequency response and transmissibility of dynamic systems under various excitation profiles.
- **CO4 (Evaluate):** Evaluate the performance of industrial vibration measurement instruments and signals against standards like ISO 10816.
- **CO5 (Create):** Design effective vibration isolation systems for industrial machinery.

Introduction to Vibration Control

- In industrial settings, heavy machinery like turbines or diesel generators produce severe dynamic forces.
- If these forces are transmitted to the foundation, they cause structural fatigue, noise, and damage.
- Conversely, delicate instruments (like an electron microscope) must be protected from ground vibrations.
- **Our Goal:** Understand the mechanics of how vibrations travel and how to decouple systems using isolation techniques.

What is Transmissibility?

Definition

Transmissibility (T_R) is a non-dimensional ratio that quantifies how much of a dynamic excitation is transmitted from one part of a system to another.

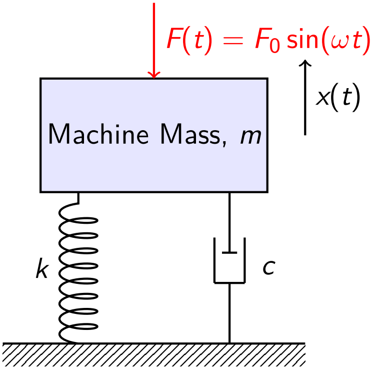
We classify this into two fundamental cases:

1. **Force Transmissibility:** The ratio of the force transmitted to the foundation to the force generated by the machine.
2. **Motion Transmissibility:** The ratio of the motion (displacement) transmitted to a machine from a vibrating foundation.

Interestingly, as we shall derive, the mathematical expression for both remains identical for linear systems.

Mathematical Model: Single Degree of Freedom (SDOF)

Let us visualize a machine mounted on a foundation via an isolator (spring k and damper c).



Force Transmissibility: Equation of Motion

Let us formulate the equation of motion using Newton's Second Law. The unbalanced force generated by the machine is $F(t) = F_0 \sin(\omega t)$.
The equation of motion is:

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin(\omega t) \tag{1}$$

The steady-state response of the mass is:

$$x_p(t) = X \sin(\omega t - \phi) \tag{2}$$

where the amplitude X is well-known:

$$X = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \tag{3}$$

Force Transmissibility: The Transmitted Force

Now, what force does the foundation actually "feel"? The foundation only feels the forces transmitted through the spring and the damper.

$$F_T(t) = kx(t) + c\dot{x}(t) \tag{4}$$

Substituting $x(t) = X\sin(\omega t - \phi)$:

$$F_T(t) = kX\sin(\omega t - \phi) + c\omega X\cos(\omega t - \phi) \tag{5}$$

Since the spring and damper forces are 90° out of phase, the maximum transmitted force F_{T0} is the vector sum:

$$F_{T0} = X\sqrt{k^2 + (c\omega)^2} \tag{6}$$

Deriving the Transmissibility Ratio (T_R)

Substituting the expression for X into our F_{T0} equation, we get the Transmissibility Ratio (T_R):

$$T_R = \frac{F_{T0}}{F_0} = \frac{\sqrt{k^2 + (c\omega)^2}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \tag{7}$$

Let us non-dimensionalize this using frequency ratio $r = \frac{\omega}{\omega_n}$ and damping ratio $\zeta = \frac{c}{2m\omega_n}$:

Fundamental Transmissibility Equation

$$T_R = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}} \tag{8}$$

This is a critical equation. Notice how T_R depends entirely on r and ζ.

Motion Transmissibility (Base Excitation)

Consider the reverse case: A sensitive instrument mounted on a vibrating floor (e.g., floor vibrating due to nearby trains).

- Base motion: $y(t) = Y \sin(\omega t)$
- Absolute mass motion: $x(t)$

The equation of motion becomes:

$$m\ddot{x} = -k(x-y) - c(\dot{x}-\dot{y}) \implies m\ddot{x} + c\dot{x} + kx = ky + c\dot{y} \tag{9}$$

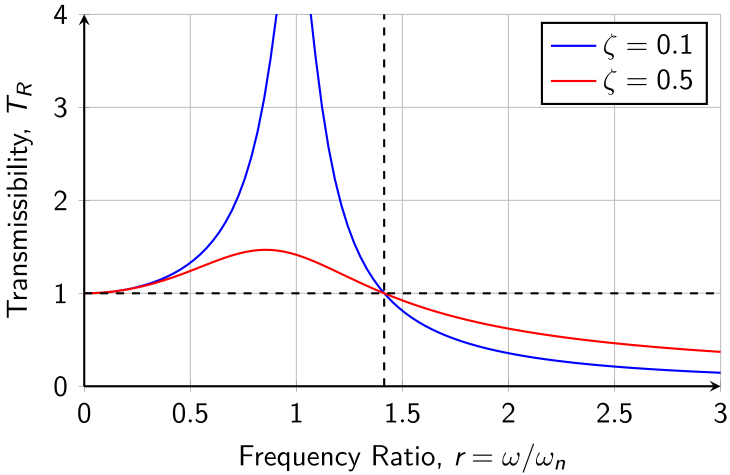
Without repeating the exhaustive algebra, solving for the ratio of response amplitude X to base amplitude Y yields:

$$\frac{X}{Y} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}} \tag{10}$$

Conclusion: Force and Motion transmissibility share the exact same mathematical behavior.

The Transmissibility Curve

Let us plot T_R versus the frequency ratio r to understand system behavior.



Interpretation of the Transmissibility Curve

Observe the graph carefully. We can divide it into three distinct regions:

1. **Region 1 ($r < 1$):** $T_R \approx 1$. The spring acts like a rigid body. No isolation is achieved.
2. **Region 2 ($r \approx 1$):** Resonance! $T_R \gg 1$. The transmitted force is highly amplified. Damping is strictly necessary here to prevent failure.
3. **Region 3 ($r > \sqrt{2}$): The Isolation Region.** Here, $T_R < 1$. The transmitted force is less than the excitation force. Notice a counter-intuitive fact: *In this region, adding damping actually worsens isolation (increases T_R).*

Design of Isolation Systems: Core Philosophy

How do we design a vibration isolation system as engineers?

- **Objective:** Keep T_R as small as possible.
- **Rule of Thumb:** We must ensure that the operating frequency ratio $r \gg \sqrt{2}$. Practically, we aim for $r \geq 3$.
- Since the operating frequency ω is usually fixed by the machine's speed (e.g., 3000 RPM motor), we must lower the natural frequency ω_n of our mount.
- Since $\omega_n = \sqrt{k/m}$, we need a *soft spring* (low k) or a *large mass* (high m, e.g., adding a concrete inertia block).

Types of Isolators in Practice

Depending on the required ω_n and the load, various materials are used:

- **Helical Steel Springs:** Offer very low stiffness (low ω_n), making them excellent for low-frequency isolation. However, they possess very little inherent damping.
- **Elastomeric/Rubber Mounts:** Commonly used in automotive engines. They provide moderate isolation and have inherent structural damping (hysteresis) to handle transient resonance during start-up.
- **Pneumatic/Air Springs:** Used for ultra-sensitive equipment (e.g., optical tables, railway coaches). Offer extremely low natural frequencies (1-2 Hz).

Numerical Example: Designing an Engine Mount

Let us solidify our understanding with a numerical problem.

Problem Statement

An industrial exhaust fan of mass $m = 200$ kg runs at a constant speed of 1500 RPM. The fan exhibits an unbalance force of 500 N. We wish to design a rubber isolation pad such that only 10% of this unbalanced force is transmitted to the concrete floor.
Assuming the damping in the rubber pad is negligible ($\zeta \approx 0$) for steady-state operation, calculate the required stiffness (k) of the isolator.

Numerical Example: Solution (Step 1)

Step 1: Identify given parameters.

- Mass, $m = 200$ kg
- Operating speed, $N = 1500$ RPM $\implies \omega = \frac{2\pi \times 1500}{60} = 157.08$ rad/s
- Desired Transmissibility, $T_R = 10\% = 0.1$
- Damping, $\zeta = 0$

Step 2: Use the Transmissibility Equation. For $\zeta = 0$, the equation simplifies heavily:

$$T_R = \left| \frac{1}{1 - r^2} \right| \tag{11}$$

Since we are in the isolation region, $r > \sqrt{2}$, so $1 - r^2$ is negative.

$$T_R = \frac{1}{r^2 - 1} \tag{12}$$

Step 3: Solve for frequency ratio r.

$$0.1 = \frac{1}{r^2 - 1} \implies r^2 - 1 = 10 \implies r^2 = 11 \implies r \approx 3.316 \tag{13}$$

Step 4: Calculate required stiffness k. We know $r = \frac{\omega}{\omega_n}$, so:

$$\omega_n = \frac{\omega}{r} = \frac{157.08}{3.316} \approx 47.37 \text{ rad/s} \tag{14}$$

Since $\omega_n = \sqrt{\frac{k}{m}}$, we find k:

$$k = m\omega_n^2 = 200 \times (47.37)^2 \approx 448,785 \text{ N/m} \approx 448.8 \text{ kN/m} \tag{15}$$

Result: The engineer must select an isolator with a stiffness of ~ 448.8 kN/m.

Vibration Measurement: Why is it required?

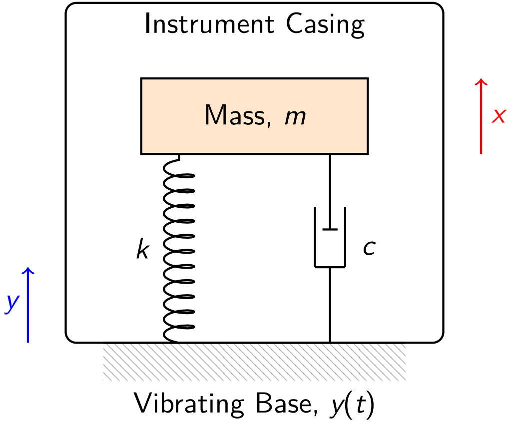
To solve a vibration problem, we must first measure it. Condition monitoring of machinery is a massive industry.

- **Preventive Maintenance:** Detecting bearing wear in power plant turbines before catastrophic failure.
- **Structural Health Monitoring:** Monitoring bridges or tall buildings during wind or seismic events.
- **Parameters to measure:**
  - Displacement (x)
  - Velocity ($\dot{x}$)
  - Acceleration ($\ddot{x}$)

How do we measure these? We use a **Seismic Instrument**.

The Generic Seismic Instrument Model

A seismic instrument consists of a mass-spring-damper system enclosed in a casing. The casing is attached to the vibrating body.



Mathematical Analysis of Seismic Instrument

The sensor cannot measure absolute motion x or y. It can only measure the **relative motion** $z = x - y$ between the mass and the casing.
The equation of motion for the internal mass is:

$$m\ddot{x} = -k(x - y) - c(\dot{x} - \dot{y}) \tag{16}$$

Substituting $x = z + y$ and $\ddot{x} = \ddot{z} + \ddot{y}$:

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \tag{17}$$

If the base harmonic motion is $y(t) = Y\sin(\omega t)$, the apparent excitation force is $m\omega^2 Y\sin(\omega t)$.

Relative Motion Response (Z)

Solving the differential equation for steady-state relative amplitude Z:

$$Z = \frac{m\omega^2 Y}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \tag{18}$$

Dividing numerator and denominator by k and introducing r and ζ:

Instrument Response Equation

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \tag{19}$$

This single equation governs both seismometers and accelerometers. The difference lies entirely in the *frequency ratio r* we design the instrument for.

Principle of the Seismometer (Vibrometer)

A **Seismometer** is designed to measure the *displacement* (Y) of the vibrating body.

- **Design Trick:** We make r very large ($r \gg 1$).
- This means ω_n must be very small. The instrument uses a very soft spring and a large mass.
- **Mathematics:** Let $r \to \infty$ in our response equation:

$$\lim_{r\to\infty} \frac{Z}{Y} = \frac{r^2}{\sqrt{(-r^2)^2}} = \frac{r^2}{r^2} \approx 1 \tag{20}$$

- **Result:** $Z \approx Y$. The relative displacement recorded by the instrument is directly equal to the amplitude of the vibrating base.

Principle of the Accelerometer

An **Accelerometer** is designed to measure the *acceleration* $(\omega^2 Y)$ of the vibrating body.

- **Design Trick:** We make r very small $(r \ll 1)$.
- This means ω_n must be very large. The instrument uses a tiny mass and a very stiff spring.
- **Mathematics:** If $r \ll 1$, then $1 - r^2 \approx 1$. Assuming small damping:

$$\frac{Z}{Y} \approx \frac{r^2}{1} = \frac{\omega^2}{\omega_n^2} \implies Z \approx \frac{1}{\omega_n^2}(\omega^2 Y) \tag{21}$$

- **Result:** Z is proportional to $(\omega^2 Y)$, which is the peak acceleration of the base. Since ω_n^2 is known, we get the acceleration!

Comparison: Seismometer vs. Accelerometer

Feature	**Seismometer (Vibrometer)**	**Accelerometer**
Measures	Displacement	Acceleration
Frequency Ratio	$r \gg 1$ (High ω)	$r \ll 1$ (Low ω)
Natural Freq (ω_n)	Very Low (Soft spring, heavy mass)	Very High (Stiff spring, light mass)
Size & Portability	Bulky, large	Very small, compact
Application	Earthquake monitoring	Engine knock sensors, aerospace, smartphones

Industry Standards: ISO 10816

In industry, we rely on standards like **ISO 10816** for evaluating machine vibration by measurements on non-rotating parts.

The Key Metric: ISO 10816 utilizes **RMS Velocity (V_{rms})** measured in **mm/s** in the frequency range of 10 Hz to 1000 Hz.

The standard categorizes machine health into four zones:

- **Zone A (Good):** Vibration of newly commissioned machines.
- **Zone B (Satisfactory):** Acceptable for unrestricted, long-term operation.
- **Zone C (Alert):** Cannot operate continuously; remedial action required.
- **Zone D (Danger):** High risk of damage. Immediate shutdown required.

ISO 10816: Machine Classifications

The acceptable velocity limits depend heavily on the size and mounting type of the machine.

- **Class I:** Individual parts of engines and machines integrally connected to the complete machine (e.g., small electric motors up to 15 kW).
- **Class II:** Medium-sized machines (15 kW to 300 kW output) without special foundations.
- **Class III:** Large prime movers and other large machines with rotating masses mounted on rigid and massive foundations.
- **Class IV:** Large prime movers and large machines mounted on foundations which are relatively soft in the direction of vibration measurement.

ISO 10816 Vibration Severity Chart

Velocity V_{rms} (mm/s)	Class I	Class II	Class III	Class IV
< 0.71	Zone A	Zone A	Zone A	Zone A
0.71 – 1.12	Zone B	Zone A	Zone A	Zone A
1.12 – 1.80	Zone C	Zone B	Zone A	Zone A
1.80 – 2.80	Zone C	Zone B	Zone B	Zone A
2.80 – 4.50	Zone D	Zone C	Zone B	Zone B
4.50 – 7.10	Zone D	Zone C	Zone C	Zone B
7.10 – 11.2	Zone D	Zone D	Zone C	Zone C
> 11.2	Zone D	Zone D	Zone D	Zone D

Note: The chart helps engineers map measured RMS velocity directly to maintenance actions based on the specific machine class.

Summary & Conclusions

To summarize today's rigorous lecture:

1. **Transmissibility** maps the transmission of force or motion. For isolation ($T_R < 1$), we strictly require the frequency ratio $r > \sqrt{2}$.
2. **Damping** limits resonance amplitude at startup, but slightly degrades isolation efficiency at high operating speeds.
3. **Seismometers** operate in the high frequency ratio regime to measure base displacement directly.
4. **Accelerometers** operate in the low frequency ratio regime, utilizing a stiff piezoelectric element to measure base acceleration.

Thank You

Any Questions?